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ELASTIC LOADING OF HIGH PRESSURE CYLINDERS

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SYNOPSIS

A thick-walled cylinder submitted to uniformly distributed internal and external pressures and to a uniformly distributed longitudinal load is considered. A graphical construction is established allowing the determination of whether the material does or does not remain elastic under this state of loads, or the selection of the value of one pressure with a view to maximizing another without the cylinder undergoing plastic deformation. Three different constructions are given corresponding to the use of the criteria of Von Mises, Tresca, and of a linearized form of the intrinsic curve of Mohr-Caquot. Several remarks on the conditions and limits in the use of this method are included.

INTRODUCTION

Notation.—The letter symbols adopted for use in this paper are defined where they first appear and are listed alphabetically in the Appendix.

Note.—Discussion open until March 1, 1965. To extend the closing date one month, a written request must be filed with the Executive Secretary, ASCE. This paper is part of the copyrighted Journal of the Engineering Mechanics Division, Proceedings of the American Society of Civil Engineers, Vol. 90, No. EM5, October, 1964.

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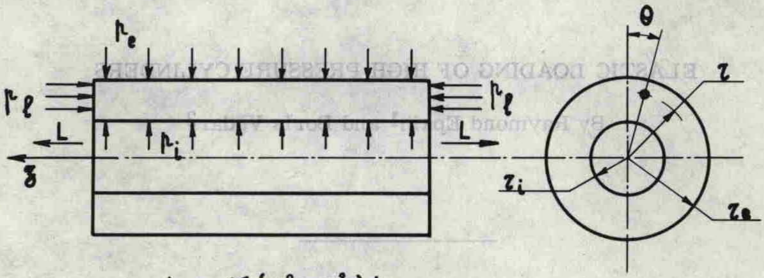
A hollow cylinder of circular cross-section, Fig. 1, is submitted to internal, p_i, external, p_e and longitudinal, p_l uniformly distributed pressures, and the limiting relations, i.e., the limit within which the vessel undergoes no plastic deformation between these quantities, are established. These relations will be referred to as elastic loading conditions and will be established for three criteria of plasticity:—the criteria of Von Mises,[3] of Mohr-Caquot[4] and of Tresca.[5]



$$L = -\pi (r_e^2 - r_i^2) p_l$$

FIG. 1

The well-known formulas of Lamé[6] give the radial, σ_r, circumferential, σ_θ, and longitudinal, σ_z, stresses as functions of p_i, p_e and p_l in the following form:

$$\sigma_r = p_i \frac{1}{k^2 - 1}\left[1 - \frac{r_i^2}{r^2} k^2\right] + p_e \frac{k^2}{k^2 - 1}\left[\frac{r_i^2}{r^2} - 1\right] \quad .. \quad (1)$$

$$\sigma_\theta = p_i \frac{1}{k^2 - 1}\left[1 + \frac{r_i^2}{r^2} k^2\right] - p_e \frac{k^2}{k^2 - 1}\left[\frac{r_i^2}{r^2} + 1\right] \quad .. \quad (2)$$

and

$$\sigma_z = - p_l \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad (3)$$

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[3] Von Mises, R., "Mechanik der festen Körper im plastisch deformablen Zustand," Göttinger Nachrichten, 1913.

[4] Caquot, A., "Définition du domaine élastique dans les corps isotropes," Proceedings, 4th Congress of Internatl. Applied Mechanics, Cambridge, Mass., 1935, p. 24.

[5] Tresca, H. E., "Mémoire sur l'écoulement des corps solides," Mémoires présentés par divers savants, Vol. 18, 1868, pp. 773-799.

[6] Lamé, G., et Clapeyron, B. P., "Mémoires sur l'équilibre intérieur des corps solides homogènes," Mémoires présentés par divers savants, 1833.